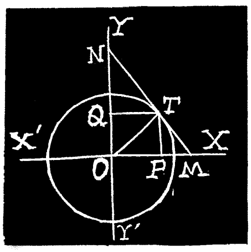
machines is one of the greatest importance to those who expect to employ Mathematics intelligently in Physics or Engineering. If we are not mistaken Professor Slichter had practically the same thought in mind when he said: "It grates on me to hear Mathematics spoken of as a tool. Mathematics is to the engineer a basal science and not a tool. The spirit of that science is of more value to the engineer than the particular things that can be accomplished. The engineer need not be a mathematician, but he needs to think mathematically, and, to my mind, he needs the power of mathematical thought more than skill in manipulating a few mathematical tools in a mechanical fashion."*

**Science*, Vol. 28 (1908), p. 263.

ON THE REPRESENTATION OF THE TRIGONOMETRIC FUNCTIONS BY LINES.

By R. D. CARMICHAEL, Alabama Presbyterian College.

The representation of the trigonometric functions dealt with in this note is in some respects different from that usually employed; it has certain advantages which appear to make it superior to other methods of effecting this representation.



Let the circle whose center is O (see figure) have the radius unity; and let O be the origin of the rectangular axes XX' and YY'. Let the angle MOT $=x$ be formed by the radius revolving counter-clockwise from OX to any position OT. Draw TP and TQ perpendicular, respectively, to OX and OY. At T draw a tangent to the circle cutting the X-axis in M and the Y-axis in N. Then in whatever quadrant OT may lie we have

$$\sin x = OQ, \quad \sec x = OM, \quad \tan x = TM,$$
$$\cos x = OP, \quad \csc x = ON, \quad \cot x = TN.$$

The tangent and the cotangent are measured from the point of tangency; the other functions from the center of the circle. Thus, all functions are measured from an extremity of the revolving radius.

It should be noticed that in any quadrant the tangent is that portion of the tangent line intercepted between the point of tangency and the X-axis, while the cotangent is intercepted between the point of tangency and the Y-axis. *This is the conception of Analytics.* The secant and the

cosine are measured along the X-axis, while the sine and the cosecant are measured along the Y-axis. The following facts are evident from an inspection of the figure for an angle in each quadrant:

The algebraic signs of the sine, cosine, secant, and cosecant are determined by the direction in which each is measured from O in accordance with the usual convention of Analytics. The algebraic sign of the tangent is plus when the tangent is measured to the right of OT (as one looks from O); minus, when measured to the left. The tangent and cotangent have always the same sign.

Any two functions of an angle measured along the same line have unity for their product.

It seems to me that this representation of the functions will make it easier for the student to fix the algebraic sign of any function in any quadrant, and also to remember the group of products each equal to unity.

The method also lends itself very readily to approximate measurements of the functions for rough work. For this purpose the pupil will require a circular protractor and a "square" graduated to tenths of the unit on the inner edges of the angle. This square should have for unit the radius of the protractor. Tangents and cotangents may be read (accurately to tenths, estimated to hundredths) by laying one inner edge of the square along the radius OT which cuts off on the protractor the required angle, and then reading TM or TN according as tangent or cotangent is desired. The sine and the cosine may be read by putting the vertex of the square as at P and reading PO and PT. (It is in measuring tangents and cotangents, of course, that this method has the advantage over the ordinary methods.)

NOTE ON AN APPROXIMATION IN TRIGONOMETRY.

By G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

The question as to whether it is possible to find the angles of a triangle in terms of the sides approximately correct, without the use of tables, is one which is often asked by persons who do not know how to use tables and yet find it necessary to use the approximate values of the angles. They understand mensuration and naturally wonder why there is not a formula for this purpose given.

The following simple deductions lead readily to such a formula.

Let A be the smallest angle of a triangle. Let the sides be denoted by a, b, c, and the area by $\triangle$. Also let $2s=a+b+c$.

Then, $\sin A=\frac{2\triangle}{bc}$, $\cos A=\frac{b^2+c^2-a^2}{2bc}$, $2+\cos A=\frac{4bc+b^2+c^2-a^2}{2bc}$,